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ABSTRACT

Seventeen research reports related to mathematics education are abstracted and analyzed. Five of the reports deal with aspects of learning theory, three with teacher education, two with testing, three with children's knowledge and understanding of various mathematics-related concepts, and one each with errors in solving linear equations, classroom behavior and cognitive development, bilingual education, and sense modality matching abilities. Research related to mathematics education which was reported in RIE and CIJE between October and December 1977 is listed. (MN)

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THE USE OF MANIPULATIVE MATERIALS AND STUDENT PERFORMANCE IN THE ENACTIVE AND ICONIC MODES. Barnett, Jeffery C.; Eastman, Phillip M. Journal for Research in Mathematics Education, v9 n2, pp94-102, March 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by John A. Dossey, Illinois State University.

1. Purpose

The purpose of this study was to test whether or not "preservice elementary teachers need actual physical experience with manipulative materials to be able to demonstrate numerical and structural properties at the concrete (enactive) and semiconcrete (iconic) levels of abstraction." While the progression through these representational modes of enactive, iconic, and symbolic knowledge has often been suggested as an efficacious pattern for mathematics instruction, its use in the training of preservice elementary teachers has not been studied.

To these ends, the following three hypotheses were stated:

H1: In measures of ability to demonstrate numerical and structural properties of the four basic arithemtic operations in the enactive mode, there are no significant differences between subjects who are required to operate in both the enactive and iconic modes and subjects who are restricted to operating in only the iconic mode.

H2: In measures of ability to demonstrate numerical and structural properties of the four basic operations in the iconic mode, there are no significant differences between subjects who are required to operate in both the enactive and iconic modes and subjects who are restricted to operating only in the iconic mode.

H3: In measures of mathematical achievement, there are no significant differences between subjects who are required to operate in both the enactive and iconic modes and subjects who are restricted to operating only in the iconic mode.

2. Rationale

Given that a large portion of the professional journals, textbooks, and activities at professional conventions are devoted to the use of manipulative materials in the teaching of mathematics, the researchers report that the amount of information on the preparation of preservice elementary teachers to use these aids effectively is both limited and contradictory.

In addition, the adoption of Bruner's representational modes at both the elementary and preservice teacher textbook levels is not based on a sound foundation of research findings. The researchers indicate that current practice, especially in the training of preservice elementary teachers, is one in which practitioners are operating on "hunches" rather than on an empirically tested base.

3. Research Design and Procedure

Two sections of a mathematics methods course for preservice elementary teachers were selected as a research population. One section was randomly designated as an experimental group and was provided with both demonstrations of how to use manipulative materials and a laboratory period during which actual "hands-on" experiences with the materials took place. The control group received the demonstrations of how to use the materials, but they received no first-hand experience with the materials. Both groups met with one of the researchers twice weekly for a one-hour lecture and then each group met with one of the researchers once a week for a two-hour laboratory session. Careful attention was paid to the equivalence of verbal information provided in both sessions.

The topics of instruction in the study period were three laboratory units on the properties of the whole number system, addition and subtraction of whole numbers, and multiplication and division of whole numbers. Students in the experimental group completed the laboratory exercises first with the concrete materials (enactive) prior to drawing a picture of the materials in the laboratory manuals (iconic). Students in the control group worked the exercises without the concrete materials, drawing pictures of what they would do (iconic) in their laboratory manuals.

The students were pre-tested with a 40-item multiple-choice content test at the beginning of the experimental period. The resulting score was used as a covariate in the analysis of the data collected. Two additional tests were given on the last day of the study. Test I consisted of 20 items similar to those covered in the laboratory manuals. Each item required the student to exhibit knowledge of a manipulative aid and a number property. Students responded to these items by drawing a picture of how the manipulatives could be used to illustrate the property. (Test I had a Cronbach alpha reliability of .72 for the groups combined.) Test II consisted of 20 multiple-choice items on the mathematical content covered in the three units studied. (Test II had a Cronbach alpha coefficient of 0.47 for the experimental group and 0.54 for the control group.)

Nineteen students from each group were then randomly selected for an interview on use of the materials in demonstrations of whole number properties. They were asked to show how they would use the materials to explain the properties to children.

4. Findings

The three hypotheses were tested using a linear regression model with the pretest used as a covariate. The only null hypothesis rejected was H3, the hypothesis dealing with the effect of the treatments on the subjects' mathematical acheivement. The results of the analysis showed that the students in the experimental (enactive and iconic) group performed significantly better than those in the control (iconic only) group ($\alpha < 0.025$).

5. Interpretations

The investigators felt that the results indicated that preservice elementary teachers do not necessarily have to interact with manipulative materials in order to function at either the enactive or iconic mode in the classroom (noted by the results on the Laboratory Test and the interview). They further stated that the Concepts Test difference is puzzling, but due perhaps to the low reliability. They also stated that the students in the experimental group may have actually learned the concepts better because of the materials, but that this difference in learning was not reflected in their ability to function in either the enactive or iconic modes.

While their study did not focus on attitudes, the researchers noted that the students in the control group seemed to have a decline in attitudes over the course of the experiment.

As a summary, they suggested that the results tend to indicate that an instructor of a preservice elementary methods class needs to weigh carefully the benefits that might be derived from the use of manipulatives against the increase in time used and the decrease in other content which will be able to be covered.

Critical Commentary

While the stated purpose of the study is both interesting and worthwhile, several problems intervene to raise serious questions about the results and interpretations made from them. Some of these concerns are:

1. Were all the tests used pre-tested on an equivalent sample of students to get estimates of their reliability? The reliability of the Concepts Test (Test II) is so low that little meaning can be associated with the related findings.

2. Working from information provided in the paper, it appears that the study involved less than 10 hours of teaching and laboratory work. Of this time, over

40 percent was spent in equivalent instruction in the classroom. This does not appear to be long enough a period of time to accrue the differences that were anticipated by the representational mode difference.

3. Were the materials used unfamiliar to members of both groups, or had the students in one or both groups seen the materials earlier in the methods course or in a prior mathematics content course? Were any checks made for this?

4. Since the items in Test I and the Interview Test were so similar to those handled in class, the failure to obtain differences here may be due to a training effect coupled with low cognitive demands in the items.

The conclusions drawn in the interpretations are far too broad to be based on the information provided in the article. While conclusions provide one view of how the data might be interpreted, one could argue that the differences on the Concepts Test are reflective of the students' internalization of the concepts. Functioning on the Concepts Test may call for a higher level of cognitive functioning than do the other two tests. This interpretation, like the one provided in the article, is also open to doubt due to the low reliability of Test II.

While the study of the role of manipulatives in the preservice education of prospective elementary teachers is an important area for study, this investigation does little to advance our knowledge in the area. It serves as a potential model of what might be done with further work on design and instrumentation.

A COMPARISON OF THREE METHODS OF INTRODUCING TWO-DIGIT NUMERATION. Barr, David C. Journal for Research in Mathematics Education, v9 n1, pp33-43, January 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert B. Ashlock, University of Maryland.

1. Purpose

The stated purpose of this study was to compare three methods of introducing two-digit numeration. The researcher sought to find out if there is any difference between experimental treatments with respect to achievement in numeration skills and application of numeration principles, both immediately after instruction and four weeks after instruction. He also hoped to identify advantages of particular treatments where differences were indicated.

2. Rationale

It is assumed in this study that children need to have a thorough understanding of the Hindu-Arabic numeration system, since this knowledge is fundamental to understanding the algorithms of arithmetic. It is further assumed that, in order to help children acquire an understanding of base ten, activities which involve grouping by tens are preferred to those which involve a more general concept of grouping.

Although most children entering school can count rationally to at least ten, experts disagree whether counting past ten should precede or follow the introduction of two-digit numerals. Therefore, research is needed which compares varied sequences for these aspects of instruction.

3. Research Design and Procedure

Children in eleven kindergarten classes from four schools in three school districts were given a battery of four pretests. Data were eliminated from the study for children who could not count a set of ten and for children who could already do tasks the treatments were designed to teach. Each remaining child was assigned to one of five cells and members of the five cells were randomly assigned to one of three experimental treatments:

- Treatment A began with the counting of sets containing more than ten members, and writing the corresponding symbols by association. Later, instruction on the meaning of the symbols were given.

- Treatment B began with grouping objects into tens and ones, and saying how many tens and ones. Children then wrote the numeral as the number of tens and the number of ones. Finally, instruction was given involving the usual number names.

- Treatment C began with the counting of sets containing more than ten members, and writing the corresponding symbols by association. Instruction then focused on grouping objects in sets of ten and counting by tens. Finally, children were taught the meaning of the symbols they had been writing.

Each treatment group received ten 20-minute lessons. Posttests were administered to test both achievement in numeration skills and application of these skills to other contexts such as the number line. In all, three maintenance lessons were given before retention tests were administered four weeks after the posttests. A MANOVA was performed on posttest and on retention test data from the three groups, with the mean treatment group score within a class as the experimental unit. A discriminant analysis was carried out on retention test data.

4. Findings

Although Treatment C obtained the highest posttest score on the criterion measure and Treatment B the lowest, The MANOVA showed no significant difference between treatments at the posttest stage.

For the retention tests, the overall difference between treatments was found to be significant ($p < .05$). The first discriminant function (which was found to be significant) gave the largest value for Treatment C. The author then stated that "Treatment C was the most effective although not necessarily significantly so."

5. Interpretations

The author notes a trend at the posttest stage which continued to the retention test stage, by which time there was a significant overall difference between treatments. The difference is attributed to achievement in numeration skills rather than application of those skills.

The relative weakness of Treatment B is interpreted as supporting the findings of Rathmell (1972). The author suggests that prior experiences of children with counting and reading number names may make it difficult for them to perceive Treatment B as something different from what they already know and can do; pre-existing counting behaviors continue unmodified.

To the author, it seems that counting more than ten objects is a well-entrenched skill as children begin school and teachers should build on that knowledge by introducing more efficient counting procedures; that is, by teaching them to count by tens as was done in Treatment C.

Critical Commentary

This study adds to a growing body of research concerned with teaching multi-digit numerals so that children have a usable understanding of what such numerals represent. At the very least, the study reminds us of two things: we must not forget there are times when careful explanatory teaching is effective and we would always do well to consider ways we can build on what children already know and can do.

The report does not indicate the results of Treatment C were significantly better than the results of either of the other treatments. Following the significant MANOVA finding, pair-wise comparisons could have been run using Hotelling's T^2. The report does not indicate this was done, yet the discussion proceeds as if pair-wise comparisons had been made; Treatment C is regarded as the "preferable" (p. 43) method. Low standard deviations are reported and one wonders if the tests are measuring what they purport to measure. Are the tests sensitive enough to measure the differences that exist?

The Rathmell study (1972) and the present study are of special interest, for the reports seem to suggest it is appropriate for children to learn more things by rote than has been assumed in recent years. In the present study, Treatments A and C both involve children writing two-digit numerals before they have learned to relate the ten's digit to sets of ten. In these treatments, children count beyond ten and learn the symbols "by association." Were the children thereby learning just by rote, or were they focusing on patterns within the sequence of oral number names and within the sequence of two-digit numerals? Ginsburg (1977) emphasizes that children find their own patterns or meanings in learning which is apparently rote. Counting from 1 through 99 is not just making 99 arbitrary associations. The child observes patterns in the sequences of names and symbols, and imputes his or her own meanings to what is observed.

The author included grouping by ten but not by other numbers, assuming that grouping experiences with other numbers are not appropriate. This assumption is based in part on the fact that Rathmell (1972) did not find significant differences. Can such non-significant differences allow us to say that one procedure is "preferable" (pp.33-4)?

The report makes clear that, before the treatments, the children could all count rationally to ten. However, it is not clear in Treatments A and C, when children counted sets of more than ten, just how far the counting actually extended. Are we to assume the numbers 11 through 99 were introduced much as we would introduce the numbers 1 through 10? This would probably include recording a numeral each time one more is joined with a set already counted, with the last number name and the numeral telling how many are in the set altogether. For Treatments A and C it is unclear, when children counted sets of 10 or more, whether they were able to do rational counting with the larger sets. Certainly, children can learn to count by rote to 100. Can the writing of a sequence of two-digit numerals be learned by a child who does not realize that when counting *any* set, the last number name tells how many are in the set? If so, would it make a difference as we plan instructional sequences?

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DESIGNING INSTRUCTIONAL METHODS IN MATHEMATICS TO ACCOMMODATE DIFFERENT PATTERNS OF APTITUDE. Becker, Jerry P.; Young, Courtney D. Jr. Journal for Research in Mathematics Education, v9 n1, pp4-19, January 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Boyd D Holtan, West Virginia University.

1. Purpose

The study investigated if two instructional treatments, Guided Discovery and Meaningful Didactic, would have an interaction effect with two aptitude variables, Reading and Reasoning.

2. Rationale

Cronbach suggested that the method of instruction or treatment may interact with the learner's individual aptitudes. Since then, researchers have been conducting ATI studies (Aptitude -Treatment-Interaction) seeking objective evidence of the interactions. Even though many teachers feel that the interaction relationships exist, researchers have not had a successful record of showing them.

3. Research Design and Procedure

This investigation used a regression analysis statistical procedure and determined the experimental groups by using a matched-pair procedure.

The subjects, 62 eighth-grade students, were selected randomly from pairs of students who had similar results on reading and reasoning pre-measures. From the "matched" pairs of students who exhibited "high reading-high reasoning", "high reading-low reasoning", "low reading-high reasoning", "low reading-low reasoning", and "middle reading-middle reasoning", subjects in each matched pair were randomly assigned to one of the two experimental treatments. The two treatments, Guided Discovery and Meaningful Didactic, were concerned with devising formulas for finding the sum of the first _n_ terms of various number series and were presented in a programmed-instruction format. The subjects worked through an introductory program to learn the terminology and symbolism and then worked through the assigned treatment.

The outcome measures were a recall of terms, notation, and formulas (Initial Learning), transfer requiring the derivation of formulas for new series (Formula Derivation), and derivation of the _n_th term formula for various series (Pattern Derivation).

Data on four other prediction variables were also collected: Time on the Introductory Program, Number of Errors on the Introductory Program, Time on the Treatment Program, and Number of Errors on the Treatment Program.

The statistical procedures were carried out in two phases. The first examined the difference of the two groups using analysis of variance. The second phase used a procedure presented by Gujarati to examine whether or not the instructional treatments interacted with the aptitude variables. Gujarati's technique (described in the 1970 American Statistician, v24, pp18-52) was used to test if regression lines and planes were parallel. If not, it was assumed that interaction existed.

4. Findings

The first phase of the analysis indicated that the two experimental groups did not differ significantly in terms of performance on the introductory and treatment programs (Guided Discovery and Meaningful Didactic).

The second phase of the analysis sought to determine, after regressing outcome scores on aptitude scores, whether the pairs of regression lines (using only one aptitude measure with two treatments) and the pairs of regression planes (using two aptitude measures with two treatments) were the same and, if not, whether significant interactions existed.

The second analysis phase showed significant interactions when time on the introductory program was used to predict Pattern Derivation and Formula Derivation. Significant interactions were also obtained when reading aptitude and errors on the introductory program were used to predict Initial Learning and when reading aptitude and time on the introductory program were used to predict Formula Derivation.

In three of the four significant interactions, time on the introductory program was an important predictor. The authors pointed out that there was one extreme time score relative to the other subjects. They repeated the analysis with this extreme score removed and the significant interactions were changed to not significant.

5. Interpretations

Using reading aptitude and reasoning aptitude as predictors, significant interactions were not found. There was some indication that time and errors on the introductory program may be of interest. The authors suggest that the treatment time may have been too short and that the predictor variables were not sufficiently error-free for the regression analysis to be appropriate.

Critical Commentary

Instructional methodology research study results have been very mixed when comparing one strategy directly with another strategy.

ATI research has been attempting to find which learning strategies and which materials might be successful with which learning attribute or style. This study was a commendable effort to refine the ATI analysis in search of possible interactions. The statistical analysis was carefully done, and the further investigation of the effect of the extreme score indicated a sincere effort to look for the interactions and to report the results.

After the first statistical procedure found no significant differences between the treatment effects (Guided Discovery and Meaningful Didactic), they quickly moved on to the second analysis. Most of the analysis was based on the predictor variable data collected during the treatments. This type of post-hoc analysis is an excellent strategy to search for hypotheses and could have led to further studies if significant interactions had been found.

Although the statistical analysis was extensive and well-handled, the problem analysis of the variables involved in the predictor measures (Reading and Reasoning) and treatments (Guided Discovery and Meaningful Didactic) might have been clarified further.

Possibly the words in the title "Designing Instructional Methods" might be a bit misleading since the study dealt more with finding ATI relationships than with designing the instructional methods.

The authors suggest that the predictor variables were not sufficiently error-free for the statistical applications used. This is quite possible. It may also be that learning and instruction is so many-faceted that collected data will be too "coarse" for the statistical procedures to be meaningful.

This study has added to the body of information on attempts to uncover ATI effects that many teachers believe to exist. Where and how to refine research further is a continuing problem.

PRESCHOOL CHILDREN'S KNOWLEDGE OF ADDITION AND SUBTRACTION. Brush, Lorelei R. Journal for Research in Mathematics Education, v9 n1, pp49-59, January 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Charles E. Lamb, The University of Texas at Austin.

1. Purpose

The purpose of the study was to investigate young children's interpretations of physical manipulations of sets of objects. In particular, two questions were asked:

(a) "How do children interpret manipulations that do contain changes in quantity?"

(b) "Are young children able to interpret correctly changes in a set through the addition or subtraction of elements?"

2. Rationale

Much of the research on children's number knowledge has concentrated on the young child's interpretation of "sameness" in quantity or amount (Piaget, 1952, and follow-up studies of conservation of number). Seldom has the focus been on the child's conception of "differences" in quantity. In many cases, children who are nonconservers will misinterpret an equality and indicate that it is a difference. This study was designed to investigate, in detail, the child's conception of "difference."

A number of studies have been done which include a sample of simple arithmetic tasks for children (e.g., Smedslund, 1966; Wohlwill, 1969). Results indicate that most children are able to identify the set with "more" objects after objects had been joined to the set. This does not mean that the child understands addition as commonly taught in schools. The terms addition and subtraction, as used by the author, deal with the manipulations of joining or taking away elements of sets. This does not imply a numerical manipulation with the sets.

The present study was designed to look at more complicated manipulations than previously dealt with in research studies. Simple tasks were altered by changing one or more features in the task. Alterations were assumed to increase the level of difficulty for the child.

Two studies are reported. The first uses nine basic tasks and the second uses additional exercises along with the basic ones to examine more carefully children's difficulties with problems of this nature.

3. Research Design and Procedure

Study 1: Twenty-six nursery school and kindergarten pupils (mean age 61.5 months) were used in the study. They were classified by sex and socioeconomic class. "To be included in the study, each child was required to pass two criterion tests. One test asked for a judgment of a static equality, the other an inequality."

Each child received nine tasks as part of a larger battery of tests. The apparatus for the tasks consisted of two cylinders. They were screened from view with the exception of the top 5 cm and were placed before the child. A one-to-one correspondence of 14 marbles was formed by dropping them one by one into the cylinders. The child sensed that there was equality in amounts of numbers. The nine tasks are indicated below:

(1) Simple Addition - one marble added to cylinder on experimenter's left.

(2) Addition Inverse - immediately following Task 1, one marble removed from the left cylinder.

(3) Simple Subtraction - following equivalence of 14 marbles, one taken from the left cylinder.

(4) Subtraction Inverse - following Task 3, one marble added to the left cylinder.

(5) Complex Addition - two marbles added to the right, one to the left.

(6) Complex Subtraction - two marbles taken from the right, one from the left.

(7) Combined Addition and Subtraction - one marble added to the right, one taken from the left.

(8) Addition and Inequality - a one-to-one correspondence of 12 marbles was established; then four added to the right, one added to the left.

(9) Subtraction and Inequality - similar to #8, then one marble taken from the right.

In each task, the child was asked, "Do both jars have the same number of marbles, or does one jar have more marbles?" The order of presentation was randomized (except that #2 always followed #1 and #4 always followed #3).

Study II: Sixty children were used in the study (20 from 4 to 5 years of age, 20 from 5 to 6 years of age, and 20 from 6 to 7 years of age). Again they were classified according to sex and socioeconomic class.

Again criterion tasks were used.

Each child received 12 items which are described below:

(1) Simple Addition: (left-hand set L) 11 flowers + 1 flower, (right-hand set R) 11 flowers.

(2) Addition Inverse: (L) 11 + 1 - 1, (R) 11.

(3) Simple Subtraction: (L) 11- 1, (R) 11.

(4) Subtract on Inverse: (L) (11 - 1) + 1, (R) 11.

(5) Complex Addition: (L) 11 marbles + 1 marble, (R) 11 + 3 marbles.

(6) Complex Subtraction: (L) 11 soldiers - 1 soldier, (R) 11 - 3 soldiers.

(7) Omitted

(8) Addition and Inequality: (L) 9 + 1 candles, (R) 13 candles.

(9) Subtraction and Inequality: (L) 9 buttons, (R) 13 - 1 buttons.

(10) Small Number Addition and Inequality: An inequality of two and six marbles was established. One marble was added to the two-marble cylinder (L).

(11) Small Number Subtraction and Inequality: Similar to Task #10, then one marble was removed from (R).

(12) Addition and Vast Inequality: An inequality of two and 15 paper clips was established. One paper clip was added to the smaller group (L).

(13) Subtraction and Vast Inequality: Similar to #12, then one paper clip was removed from (R).

Children were asked three short questions for all situations:

(1) "Do we have the same number of _____?"

(2) "Does one of us have more _____?"; and

(3) "Why is that?"

Tasks were presented randomly with certain restraints:

(1) #2 followed #1; (2) #4 followed #3; (3) #10-13 were given

to children who missed either #8 or #9; and (4) #10-13 were given in that order at the end of other tasks.

4. Findings

Study I:

(1) No sex or socioeconomic differences were observed in performances on tasks. A majority of the children got all tasks correct.

(2) Tasks 1, 3, 5, and 7 seemed very easy (over 90% of subjects correctly responded).

(3) Complex Subtraction was more difficult than Simple Subtraction or Complex Addition.

(4) Inequality tasks were more difficult than tasks involving only the simple operations.

(5) Inverse tasks were somewhat more difficult than tasks using simple operations.

Study II:

(1) Again, there was no difference with regard to sex or socioeconomic class.

(2) Tasks 1 and 3 were very easy.

(3) Inverse and Inequality tasks were more difficult than tasks involving simple operations.

(4) Subtraction seemed more difficult than addition.

(5) Performance increased with age.

(6) Children seem to be able to cope with initial inequalities when the initial sets are small or if differences are quite large.

5. Interpretations

Study I:

(1) Children do surprisingly well on tasks involving simple arithmetic operations.

(2) The author feels that most children will make the transition from tasks of this sort to numerical problems with little difficulty.

(3) Most children recognize the inverse relationship between the adding and subtracting operations.

(4) The author suggests use of these tasks by teachers as a diagnostic tool, with particular attention paid to the child's use or nonuse of numerical ideas.

(5) Because of children's sophistication with these ideas, the author suggests that teachers use the child's basic knowledge to build mathematical knowledge.

Critical Commentary

(1) The research is in an important area. It examines in a detailed way some very basic arithmetic ideas which children have. The use of one-on-one interviews is commendable.

(2) The statement that "most recognize the inverse relationship of these two operations" appears to contradict other results in the related literature. More discussion of this point would have been helpful. Perhaps the differences are due to task differences.

(3) "For example, if a child spontaneously uses numbers in answering these tasks, the teacher is assured that the child has a solid basis of knowledge on which more formal arithmetic structures can be built" is a statement that seems a bit strong. The spontaneous use of numerical information may indicate confusion rather than understanding.

(4) Studies of this sort are very important to the further understanding of the young child's number conceptions. Unfortunately, the results are often generalized too far. These results, when analyzed and synthesized with others, should prove very fruitful.

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LOCATION OF A POINT IN EUCLIDIAN SPACE BY CHILDREN IN GRADES ONE THROUGH SIX. Carlson, G. R. Journal of Research in Science Teaching, v13 n4, pp331-336, 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Carol A. Thornton, Illinois State University.

1. Purpose

(a) To use Piagetian-type tasks to examine the ability of a sample of children to locate quantitatively a point in one, two, and three dimensions.

(b) To examine a small part of Piaget's theory of cognitive development.

2. Rationale

Piaget's theory of cognitive development forms a theoretical base that may be related to science and science instruction. This study was designed and is interpreted according to the framework Piaget provides in exploring children's understanding of Euclidean space. The specific abilities examined in the study are needed in such scientifically and mathematically related activities as quantitative linear measurement (as opposed to qualitative comparisons of lengths) and precise two-dimensional graph construction and interpretation.

3. Research Design and Procedure

The population for the study was drawn from grades one through six of a midwest elementary school. In order to aid in identifying any age level(s) at which subjects would show a marked change in ability with respect to any of the tasks, the population at each grade level was stratified and separated from adjacent grade levels by an age span of six months. For the sample, eight males and eight females were randomly selected from each grade level--a total of 96 subjects.

The three tasks posed to subjects of the study were basically the standard Piagetian tasks for exact location of a point in one, in two, and in three dimensions (tasks 1, 2, and 3 respectively). Task 1 involved two strings stretched tightly between two nails. The two strings were the same length and parallel, but were staggered so that endpoints were not aligned. As a subject watched, the experimenter moved the bead on the left end of one string 14 centimeters to the right. The subject was then asked to move the bead on the left end of the other string just as far, using any of the unmarked measuring instruments which were available. (One was the same length, one longer, and one shorter than 14 cm.)

In Task 2 the subject was shown two cards placed in opposite corners of a larger card. The experimenter placed a dot on one of the cards and then asked the subject to mark his or her card in the same place. Potential measuring instruments and a pencil were provided.

In Task 3 the subject was shown two skeletal cubes depicting identical small rooms. The experimenter pointed out a small clay ball (simulating a light bulb) which hung from the ceiling of one of the "rooms". The subject was asked to put an identical clay ball in the same place in his or her room. Several rods of different lengths and other potential measuring instruments were provided.

Subjects were presented all three tasks on a one-to-one basis over a six-week period. Each interview required 20-25 minutes. Piaget's three scoring stages were used to evaluate children's responses: Stage I--child did not understand the task or the principles involved; Stage II--child is in a transitional stage, groping for the right solution; Stage III--child, by correct response and explanation, demonstrated a steady understanding of concepts involved. In addition, substages were used to assess more accurately children's thinking. Subjects were also scored on a pass (reached Stage III) or fail basis.

A three-factor ANOVA was used to analyze overall task performance for the six grade levels. A comparison was also made, within subjects, of performances on the one-, two- and three-dimensional tasks. A chi-square analysis was performed to assess the relationship between sex and task performance.

4. Findings

The analysis of variance revealed a significant difference in the overall task performance (one Tasks 1, 2, and 3 collectively) between the six grade levels. Subjects did progressively and continuously better in their overall performance from grades one through six.

On the specific task analysis, the performance of subjects was found to differ significantly ($p < .05$). One-dimensional tasks were easier than the two-dimensional tasks, and these in turn were easier than the three-dimensional tasks. No sex-linked differences were found in the chi-square analysis of task performance by females and by males.

5. Interpretations

(a) The results of the study indicate a difference between a subject's ability to locate a point in two and in three dimensions. This conclusion disagrees with Piaget's data and theory, in which no time-lag whatever existed in the achievement of successive levels of two- and three-dimensional measurement.

(b) Contrary to Piaget's findings, in which a <u>decisive turning point</u> in subjects' development of spatial concepts related to these tasks occurred (at about age nine), the data of the present study relative to subjects' performances on two- and three- dimensional tasks indicate a smooth and continuous increase in achievement.

(c) The following remarks are based on analysis in which 75 percent or more subjects passed the task. This 75 percent criterion has previously been used by Piaget to analyze group performance.

1) Activities requiring only qualitative measurement (defined by Piaget as the use of a measuring instrument which is longer or the same length as the distance to be measured) would probably be meaningful to these subjects by about the second-grade level. Prior to this level these subjects have only a knowledge of which of two objects is the longer and no measurement can be made. Activities such as strip-chart graphing, however, showing the relative or comparative growth of two plants, would be meaningful to most children by first grade. (Based on analyses of Task 1)

2) Science-related activities requiring students to make a line graph, to locate points on a map, or to make precise geometrical drawings may not be meaningful to these subjects until the fourth grade. (Based on Task 2 analyses)

3) Activities requiring the subjects to visualize precise quantitative three-dimensional shapes or to make, for example, a quantitative comparison of the location of an airplane in flight with respect to its point of destination may not be meaningful until approximately the sixth-grade level. (Based on analyses of Task 3)

Critical Commentary

Replication studies, such as this one, are important in their own right. Both from the perspective that much of Piaget's research has direct implication for teaching that involves mathematics or mathematically-based science tasks, and that Piaget worked with children culturally different from our own, it is appropriate to duplicate his studies.

That the present study was carefully designed and executed is evident from several standpoints; to be noted in particular is the development of stratification procedures prior to sampling and the adherence to protocol during the interviews. The cautious, yet insightful, interpretation of the data is both educationally interesting and informative.

The biggest problem in reviewing the research lay in the logic, clarity, and completeness of the research report. In describing the methods and procedures, for example, it would appear that the general design of the three tasks used in the study should appear before the layout of administration and scoring procedures. At times the writing style is awkward or unclear. Several important sources or items of information, if included, would have strengthened the overall clarity of the data analysis: the ANOVA table, commentaries on tabled information which did appear, and a report of means and standard deviations. Post hoc analyses in case of an overall significant F ratio might also have been used and reported to support differences in specific comparisons. Apart from these considerations the research itself and the implications drawn are both important and valuable.

IDENTIFYING ERRORS IN SOLVING CERTAIN LINEAR EQUATIONS. Davis, E. J.; Cooney, T. J. Mathematics Association of Two-Year Colleges Journal, v2 n3, pp170-178, Fall 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Bert Waits, The Ohio State University.

1. Purpose

To investigate the type and frequency of errors commonly made by algebra students in solving linear equations.

2. Rationale

Teachers need to be knowledgeable about the types of difficulties students encounter when solving linear equations (1) in order to teach the topic effectively and (2) to help the individual student overcome a particular difficulty.

3. Researc sign and Procedure

Approximately 110 "second-year basic algebra" and "regular algebra" students were given a 12-question examination designed by the authors to test a wide range of manipulations involved in the solution of linear equations. The students used in the investigation were thought to be no different from other algebra students from the same high school. The students' teachers stated that they were prepared for the examination. The students were told their "performance would be recorded."

The authors identified 11 categories of errors:

1A. Misuses of the rules for adding or subtracting positive and negative numbers

1B Arithmetic errors in addition or subtraction

2A Misused rules for multiplying or dividing positive and negative numbers

2B Arithmetic errors in multiplying or dividing

3A Misuse of the additive property of equality--this category includes "transposing" errors

3B Misuses of the multiplicative property of equality

4A Coefficient error--uses additive rather than multiplicative inverse

4B Coefficient error--misuses multiplicative inverse

5A Does not finish--original equation contains fractions

5B Does not finish--original equation does not contain fractions

6 Others (the majority of these were miscopying errors and undecipherable solutions).

The authors graded and analyzed each test and recorded the types of errors made.

4. Findings

The authors' findings, consisting of a frequency analysis of errors by category, were presented in several ways. The last two tables indicate the frequency of error by category for "good" students and for "poor" students.

Table 1

Number and Relative Percent of Errors in Each Category for Regular and Basic Algebra Students

	1A	1B	2A	2B	3A	3B	4A	4B	5A	5B	6	Total
72 Regular Algebra Students	68 26%	8 3%	21 9%	41 16%	11 4%	27 10%	12 5%	8 3%	25 10%	5 2%	34 13%	260
38 Basic Algebra Students	60 32%	10 5%	13 7%	21 11%	3 2%	18 10%	9 5%	10 5%	26 14%	1 1%	17 9%	188
Total for 110 Students	128 29%	18 4%	34 8%	62 14%	14 3%	45 10%	21 5%	18 4%	51 11%	6 1%	51 11%	488

Table 2

Errors by Category for Students
Getting 10 or 11 Equations Correct

	1A	1B	2A	2B	3A	3B	4A	4B	5A	5B	6	Totals
26 Regular Algebra Students	15	0	4	14	2	2	1	1	4	0	6	49
9 Basic Algebra Students	10	1	3	5	0	0	1	0	0	0	0	20
Total for 35 Students	25	1	7	19	2	2	2	1	4	0	6	69

Table 3

Errors by Category for Students Getting
only 2, 3, 4, 5, 6, or 7 Equations Correct

	1A	1B	2A	2B	3A	3B	4A	4B	5A	5B	6	Totals
21 Regular Algebra Students	45	7	10	24	18	8	10	6	17	6	26	117
18 Basic Algebra Students	49	8	7	14	16	2	5	10	25	1	12	149
Total for 39 Students	94	15	17	38	34	10	15	16	42	7	38	326

5. Interpretations

The authors discussed the following question:

(a) How well did the students do on the inventory?

Very Well: Mean 8.69, Median 10, and Range 2 to 1[illegible]

(b) To what extent were errors computational in nature?

Considerable: At least 54% of the errors were category 1 or 2.

(c) To what extent were errors attributable to an inability to apply principles for solving equations correctly?

22% of the errors were in categories 3 and 4.

(d) To what extent were students unable to complete the process of solving an equation?

There were 57 cases (the percent was not stated) of category 5 errors; 51 of them involved equations with fractions (e.g., $\frac{3}{2}x = \frac{5}{7}$), while only 5 (of 12) problems involved fractions.

(e) Did students who scored high make different types of errors from students who scored low?

Most errors (75%) of the "good" students (those with 10 or 11 correct solutions) were computational, while 50% of the errors of the "poor" students (those with 2 to 7 correct solutions) were computational.

In conclusion, the authors point out that their analysis could have other positive benefits in addition to the diagnostic assistance given to the individual student. It can be used with the entire class to discuss the types of errors made (and ways to correct them). They note that perhaps such a class discussion can help the individual student avoid similar pitfalls.

Critical Commentary

This investigation is interesting but limited in scope and subject to subjective interpretation. The authors admit that their error categorization system is "but one of many systems that could be developed and used."

Are there better error analysis systems?

Is their system valid?

What constitutes a valid error analysis system?

Are the authors' judgments about types of errors appropriate?

These are difficult questions. Indeed, there are no "correct" answers.

No previous research was cited. Has there been similar research conducted on the same or related topics?

It was stated that the students were from a Georgia high school. What was the nature of "second-year basic algebra"? Was it remedial? No effort was made to analyze the difference, if any, between the two groups of algebra students. Tables 2 and 3 would be enhanced considerably if percentages had been given (as in the first table).

The authors have objectively quantified student errors for one type of elementary algebra task. However, is their system practical for the typical classroom teacher with 120-150 students and many algebra tasks to teach?

It is clearly desirable that teachers know what kind of errors their students make so that steps can be taken to correct the errors (both in teaching strategies and in directing relearning by the individual student). Perhaps the authors' ideas can help the classroom teacher come up with a more efficient system (albeit more subjective) to analyze and resolve students' errors.

PRETRAINING CHICANO STUDENTS BEFORE ADMINISTRATION OF A MATHEMATICS PREDICTOR TEST. Ginther, Joan R. Journal for Research in Mathematics Education, v9 n2, pp118-125, March 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by C. James Lovett, Brooklyn College of the City University of New York.

1. Purpose

The purpose of this study was to determine if the reliability and predictive power of certain mathematics predictor tests given to Chicano students could be increased by providing pretraining in test-marking strategies before administration of the tests.

2. Rationale

Earlier studies have found that ability tests that are good predictors of mathematics achievement for Anglo students are not generally good predictors for Chicano students. Bernal (1971) found that teaching Chicano students test-marking strategies improved their results on mental ability tests, but the report did not include the effects, if any, on the reliability or predictive power of the tests. The present study was designed to investigate the possibility of such effects.

3. Research Design and Procedure

The subjects were 136 students from the seventh-grade population of a junior high school selected because it had a large Chicano population. Students for whom English was a second language were excluded from the study. Students were classified as Chicano if they had a Spanish surname. On this basis, 44 percent of the sample was Chicano.

On a classroom basis, the sample was randomly divided into two groups. One group received pretraining in a test-marking strategy for an Arithmetic Reasoning Test. The other group received similar pretraining for a Missing Words Test. In both cases, the 30-minute training session consisted of an explanation/discussion of the strategy and practice on sample items. Both tests were administered to the entire sample. Over a five-day period of time, the students then studied a set of programmed instructional materials and were administered a posttest related to the content of the materials. Descriptions of the test-marking strategies, sources for all tests and materials, and a detailed schedule for each phase of the study are included in the original report.

A retention study is also reported. The above experiment was conducted in June 1975; in September both predictor tests were administered again.

4. Findings

The sample was divided into four groups according to whether students were Chicano (C) or non-Chicano (NC) and whether they were pretrained for the Arithmetic Reasoning Test (AR) or the Missing Words Test (MW). For each group a reliability coefficient (Cronbach's Alpha) was computed for each predictor test. On the basis of tabular data, it was noted that pretraining improved the reliability of the Arithmetic Reasoning Test for Chicano students, and that this effect was maintained for the second testing (retention study). Similar effects were not obtained for the non-Chicano group on the Arithmetic Reasoning Test or for either group on the Missing Words Test.

The predictive power of a test was defined as its correlation with the posttest. The results are summarized below.

Group	Arithmetic Reasoning	Missing Words
C-AR	.67	.55
C-MW	.40	.72
NC-AR	.53	.31
NC-MW	.49	.52

The difference between the correlation coefficients for each pretraining/no pretraining condition was tested for significance for Chicano and non-Chicano groups. For the Arithmetic Reasoning Test the difference between the pretrained (C-AR) and the not pretrained (C-MW) Chicano groups was significant at the .08 level. For the Missing Words Test, the difference between the pretrained (-MW) and not pretrained (-AR) groups was significant at the .15 level for the Chicano students and at the .14 level for the non-Chicano students. For the Chicano groups, the probability that both increases in predictive power took place simultaneously by chance is less than .012 (.08 x .15).

5. Interpretations

The pretraining improved the reliability and predictive power of the Arithmetic Reasoning Test for the Chicano students, but it had no effect for the non-Chicano students. For the Missing Words Test, pretraining had no effect on reliability for either group; it had a positive effect, however, on the test's predictive power for both Chicano and non-Chicano students. The results tend to confirm those of Bernal (1971).

The author calls for replications of her study using a variety of other mathematics predictor tests. If the replications obtain similar results, then it is recommended that testing of Chicano students be preceded by pretraining.

Critical Commentary

This type of research is of interest to many people in the area of bilingual education. This includes school officials responsible for the placement of children as well as researchers. The author appears to have conducted a very careful study, especially in terms of the procedures employed. There are several points, however, which I feel must be raised.

The use of Spanish surname as the defining criterion for "Chicano" implicitly assumes that this classification yields a relatively homogeneous group with respect to variables relevant to school learning. Such an assumption is open to challenge even for a sample drawn from a single school (see Ramírez and Castañeda, 1974). Furthermore, as written, the report suggests that Chicano students with English as a second language were excluded from the study. If this was indeed the case, I begin to wonder about the theoretical basis of the study. How were Chicano and non-Chicano groups different other than by surname?

There are several points where I must disagree with the author's treatment and interpretation of the data. First, the author claims that the retention study of test reliability supports the initial conclusion of treatment effect for the Chicano group. The data presented, however, show that over the summer months, with no further treatments, the reliabilities changed by an amount larger than the original difference between the two groups (from .57 to .72 for one group and from .26 to .08 for the other). Such changes suggest the possibility of factors other than the maintenance of a treatment effect.

In another matter, I may be old-fashioned, but I cannot agree with the author's style of reporting significant differences. If authors choose not to use the traditional .05 level for decisions regarding significance, then I think they should tell us at just what point they no longer consider a difference to be significant. I must also take issue with claims based on multiplying the probabilities found for various differences. This approach yields differences tested, and I do not see that any useful information is gained.

Finally, there is an aspect of the data not mentioned by the author, but which I think should be noted. The study is based on the supposition that tests that are good for Anglo students are not so for Chicano students. Yet, in the table provided we see that the predictive power of the Missing Words Test is greater for the not pretrained Chicano group (.55) than for the not pretrained non-Chicano group (.31). Even for the Arithmetic Reasoning Test the difference between the not pretrained Chicano (.40) and non-Chicano (.49) groups is not large. These results, which contradict the earlier studies cited by the author, are not discussed or even noted.

In conclusion, I agree that replications are needed, but I would urge researchers to give serious consideration to the points mentioned above before initiating such studies.

References

Bernal, E. M. Jr. Concept Learning Among Anglo, Black, and Mexican-American Children Using Facilitation Strategies and Bilingual Techniques (Doctoral Dissertation, University of Texas at Austin, 1971). Dissertation Abstracts International, v32, 6180A-6181A, 1972. (University Microfilms No. 72-15,707)

Ramírez, M. and Castañeda, A. Cultural Democracy, Bicognitive Development, and Education. New York: Academic Press, Inc., 1974.

RELATIONSHIPS BETWEEN CLASSROOM BEHAVIOR AND COGNITIVE DEVELOPMENT CHARACTERISTICS IN ELEMENTARY SCHOOL CHILDREN. Good, Ron; Matthews, Charles; Shymansky, James; Penick, John. Journal of Research in Science Teaching, v13 n6, pp533-538, November 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Karen Fuson, Northwestern University.

1. Purpose

This study examined the relationship between the performance of students in grades 1 through 5 on seven conservation tasks in nine categories of science classroom behavior. Two different types of science classrooms were examined--student-structured and teacher-structured.

2. Rationale

This study was an attempt to relate some student cognitive characteristics as measured by Piagetian conservation tasks to some classroom learning behaviors. The basic questions of the study were: "Do conservers behave differently from non-conservers in science classrooms?" and "Are differences constant across different classroom environments?"

3. Research Design and Procedure

All of the students in the first five grades at Florida State University's Developmental Research School comprised the sample. Students are selected for the school to represent the population at large (the nature of the population at large was not specified). Students were randomly assigned to the TSLS (Teacher Structured Learning in Science) or the SSLS (Student Structured Learning in Science) treatments. Piagetian conservation tasks of the following types were given to each student at the beginning of the year and at the end of 30 weeks: number, area, weight, displacement, perimeter, perimeter-area, and internal volume. Student behaviors on a set of nine categories were coded throughout the year (number of times unspecified.) However, only the post-test scores and a coding done near the end of the 30 weeks were used for this study. One-way analysis of variance was used to determine, within each type of classroom, whether conservers and non-conservers differed on any of the nine behavioral categories.

4. Findings

1. In a teacher-structured but not in a student-structured setting, non-conservers observed the teacher significantly more often than did conservers.

2. In a teacher-structured but not in a student-structured setting, conservers followed teacher's directions significantly more often than did non-conservers.

3. In a student-structured but not in a teacher-structured setting, conservers tended to make up their own activities with sets of manipulative materials significantly more often then did non-conservers.

4. No significant differences in either type of setting existed for the following categories: responds to teacher question, initiates interaction with teacher, initiates interaction with another student, receives ideas from another student, copies another student, gives ideas to another student.

5. Interpretations

The authors report the following interpretations of their findings:

> If a science curriculum provides each child with an opportunity to work with manipulative materials which offer a wide range of possible activities, then the "less cognitively advanced" children in a classroom are more susceptible to the influence of certain teacher behaviors than are "more cognitively advanced" children. Teacher's directions, evaluations, etc., seem to divert the non-conservers' attention away from more productive activities with science materials to a greater degree than for conservers. Because of this, a TSLS strategy could work to the disadvantage of the "slower" students in a classroom even though they may have the most to gain from a science class rich with manipulative materials.

Critical Commentary

The objective of this study was a noteworthy one. Relatively few studies exist which relate student performance on Piagetian tasks to student school outcome measures and even fewer which relate such performance to student classroom behavior. The attack on the basic problem seems to have been a fairly comprehensive one. However, because classroom behavior can be considered as a variable intervening between cognitive abilities and achievement, the design would have been stronger if it had also included measures of student learning in the science class. Some examination of differences across grade level might also have been done.

Unfortunately, the study was not reported well. No mean scores were given in the entire paper, so one has no way to judge what the order of magnitude of the difference in behavior was. The text contains some irritating errors: one of the categories is omitted from Table 1 and "non-conservers" is used in the place of "conservers" in reporting one of the major results. Results are reported and discussed much less specifically than one should expect, and some important findings (such as differences between the cognitive tasks) are not discussed at all. Giving the numbers of conservers and non-conservers at each grade level would have been informative. A two-way analysis of variance using conserving ability and type of classroom would have been a more appropriate analysis. This study was basically well-designed, and the coding of student classroom behavior and the Piagetian interviews of 250 children represent major investments of time and energy. However, little can be made of the results because of inadequate analysis and reporting.

The implications the authors draw from their results deserve special mention (see the Interpretations section above). The results as reported merely say that in teacher-structured classrooms, non-conservers observe the teacher more and conservers follow the teacher's directions more. It is not clear whether this difference arises because the non-conservers cannot follow the teacher's directions and must be shown the activities, whether the teacher is demonstrating for everyone and the conservers simply do not pay attention, or whether the teachers give in to the temptation to do an activity for the non-conservers because it is easier than helping them do it themselves. It may be that the less advanced students can learn more from watching a well-organized and clearly explained experiment than they can by muddling through a little-understood procedure themselves. It is unclear whether the authors' inference that the teacher diverts non-conservers' attention "away from more productive activities with science materials" stems from a value that activity in itself is productive and good or from some unreported data (or even intuitions from observations) about the non-learning nature of the non-concerver interactions with the teacher. If it is the latter, then the authors should have reported that additional data to substantiate their claim. That a teacher-structured classroom can work to the detriment of slower students would be an important result (though achievement data would be useful in answering this question). However, the data as reported do not permit this inference to be drawn.

Further, it is not clear that conservation ability is <u>causally</u> related to the dependent measures at all. The cognitive characteristics as measured by the conservation interviews may be "influential in determining" interactions, or some other student characteristic(s) correlated with those cognitive characteristics may be the actual "cause". And, in fact, the pre-test conservation measures would logically seem to be more likely causes of student behavior which "stabilized early in the year" than would the post-test measures.

One note of interest in this study not mentioned by the authors is that the significant differences in classroom behaviors of the conservers and non-conservers were reflected more on the area, weight, perimeter, and internal volume tasks than on the number, displacement, and perimeter-area tasks. Thus, in future research, one might wish to focus only on these tasks. These task differences may, however, partially result from the age range studied (e.g., number conservation may have reached a ceiling). Again, some reporting of possible effects of grade level would have been informative.

In summary, one can conclude very little from this article. If the article had been more carefully reviewed before publication, some of the major shortcomings in the reporting of the results might have been corrected, and the study might have contributed to our knowledge in this area.

THE EFFECTS OF GUIDED DISCOVERY AND INDIVIDUALIZED INSTRUCTIONAL PACKAGES ON INITIAL LEARNING, TRANSFER, AND RETENTION IN SECOND-YEAR ALGEBRA. Hirsch, Christian R. Journal for Research in Mathematics Education, v8 n5, pp359-368, November 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Aaron D. Buchanan, SWRL Educational Research and Development, Los Alamitos, California.

1. Purpose

To see which of three modes of instruction would be most beneficial in promoting: (1) initial learning of elements of structure in the system of complex numbers, (2) vertical and lateral transfer measured at the same time as initial learning, and (3) retention of initial learning measured six months later. The three modes of instruction were:

(a) guided discovery, where learning was guided by a teacher using special techniques to promote discovery

(b) expository individualized instruction, where students went through a special learning package written in linear format

(c) programmed individualized instruction, where students went through a special learning package written in branching format.

2. Rationale

Experts in mathematics education have advocated that instruction be presented in such a way that pupils can discover elements of mathematical structure for themselves. However, techniques for promoting discovery learning have not been used very extensively in traditional teacher-centered instruction. For this reason, individualized instructional packages have often been used as one way to counteract the persistent use of traditional teaching methods which ostensibly inhibit discovery learning.

3. Research Design and Procedures

The basic design of the study involved three treatments followed by a series of tests (posttest, lateral transfer, vertical transfer, and long-term retention) to measure differences in treatment effects. Eleven algebra II classes were assigned as entire classes to the three treatments. Four classes were given the guided discovery treatment, four classes were given an expository individualized instruction package, and three classes were given a programmed individualized instructional package. There is some indication that groups were assigned to treatments based partly on whether or not they had previously been involved with individualized instruction packages. The investigator was concerned that Ss would have difficulty in adjusting to this type of instruction within a short time frame, and that it might result in diminished treatment effects on student performance.

Two standardized algebra tests were given at the beginning of the study so that measures of treatment effects could be adjusted for what turned out to be small pre-treatment differences between experimental groups. In addition, all classes had completed the same amount of their algebra II course work prior to the study.

The investigator and two regular classroom teachers were responsible for the guided discovery treatment. The investigator would teach a demonstration lesson to one guided discovery class each day which the two regular classroom teachers would observe and then repeat with their own guided discovery classes. Content for the guided discovery sessions was taken directly from the expository individualized instruction package. Classes receiving one of the two different kinds of individualized instruction worked through learning packages on their own.

Each treatment dealt with a common set of mathematical concepts drawn from the structure of complex numbers. Special care was taken to insure that guided discovery classes did not go beyond the scope of content in the individualized instruction packages. Each treatment lasted six days. Guided discovery classes spent six regular class periods in instruction; Ss receiving individualized instruction completed six one-day learning packages that were either expository or programmed.

4. Findings

Two-hundred thirteen Ss were treated as though they had been assigned separately to one of the three treatments. The analysis involved 61 Ss in guided discovery, 84 Ss who were given the expository individualized instruction package, and 68 Ss who were given the programmed individualized instruction package.

A test of initial learning (25 items) was constructed by the investigator to reflect content in the three treatments. On this test, Ss in guided discovery scored significantly higher ($p<.01$) than Ss completing either of the individualized instruction packages.

The vertical transfer test (11 problems worth 5 points each) included topics that were more advanced (e.g., functions of complex variables) than students' initial learning, but ostensibly required no more specific knowledge than what pupils had already developed. The only significant differences in performance were associated with the guided discovery treatment.

The lateral transfer test (7 problems worth 5 points each) required pupils to generalize elements of structure they had learned about complex numbers to other mathematical systems with similar structural characteristics. Again Ss in guided discovery scored significantly higher than Ss completing either of the individualized instruction packages.

On all three tests given immediately after the treatments, there were no significant differences in performances attributable to the two different types of individualized instruction packages. On the retention test given six weeks later, there were no significant differences attributable to any treatment. Ss in guided discovery did score higher than Ss who had received one of the individualized instruction packages, but mean scores on the retention test were all very low.

5. Interpretations

Students guided by a teacher to discover elements of structure in the system of complex numbers performed significantly better than students who covered the same content on their own in individualized instruction packages. Not only did guided discovery result in a higher level of initial learning, but this learning transferred better to other kinds of related learning tasks.

Critical Commentary

The results of this study have most significance in showing better effects from teacher-guided instruction than from self-guided packages in the learning of new content. It is not possible to say with much precision why teacher-guided instruction was better. The study was not designed to show the effects of "discovery" teaching as distinct from teacher-guided versus self-guided instruction. The rationale gives a limited amount of support for the actual experiment as it was conducted. The investigator tried to establish a relationship between individualized learning packages, discovery learning, and the concept of "active" learning, but this relationship was not made very clear. In general, the rationale was tied in only the broadest way to other research on mathematics instruction. No specific research studies were cited.

The investigator did appear to maintain very close control over content presented in each of the three treatments, which is always difficult in this kind of classroom-based study. Therefore, it is possible to infer some advantage for teacher-guided instruction. It is reinforcing to see results that show significant differences for different kinds of instruction, especially when so many other studies show no significant differences at all. Now, if we could only make better inferences about why such differences occur. . .

STRUCTURA... LINGUISTIC VARIABLES IN SELECTED INFERENCE PATTERNS FOR BILINGUAL... GRADES SIX TO TEN. Jurdak, M. E. Educational Studies in Mathem tics, v8 n2, pp225-238, August 1977.

Expanded A tract and Analysis Prepared Especially for I.M.E. by Thomas J. C...ney, University of Georgia.

1. Purpose

The investigator sought to identify the relative importance of structural and linguistic variables in assessing students' ability to deal with inference patterns and to investigate whether different structural variables were associated with the native (Arabic) or foreign language (English) used for bilingual subjects.

2. Rationale

Previous research has focused on the nature of students' errors in using inference patterns and on the identification of structural and linguistic variables that account for the variance in students' performances. The present study represents a continuation of this line of research.

3. Research Design and Procedure

Test. The test consisted of 64 items based on four sequences (two of propositions and two of propositional functions) as i icated below:

If Samir's car is a police car, then it is black.

If Samir's car is black, then it is comfortable.

If Samir's book is an arithmetic book, then it is new.

If Samir's book is a new book, then it contains beautiful pictures.

All the new books contain beautiful pictures.

All the arithmetic books are new.

All police cars are black.

All black cars are comfortable.

Format 1

Premise	Premise	Premise	Question	Responses
$(p) \rightarrow (q)$	$(q) \rightarrow (r)$	(p) is true	Is (q) true?	yes,
			or	no,
			Is $q(x)$ true for all x?	maybe yes, maybe no

Format 2

Premise	Premise	Premise	Question	Responses
$\forall x(p(x) \rightarrow q(x))$	$\forall x(q(x) \rightarrow r(x))$	$p(x)$ is true	Is $q(x_o)$ true?	yes,
			or	no,
			Is $q(x)$ true for all x?	maybe yes, maybe no

Variables. Four structural variables were defined:

Decidability (D): An item was decidable if a yes or no response could be given rather than maybe yes or maybe no.

Type of Logic (L): An item was categorized as belonging either to the propositional calculus or to the predicate calculus.

Length of Sequence (S): The sequence length of an item was determined by whether there were one or two propositions (or propositional functions) involved.

Overgeneralization (O): An item had overgeneralization if the premises were propositions and the questions were about the corresponding propositional functions.

There were 12 linguistic variables in English and 13 linguistic variables in Arabic. In general, these variables involved the number of occurrences of such grammatical components as nouns, verbs, articles, and predicates.

Three other variables used were: Existence of 'all' in the conclusion (A); Negative occurrence of 'all' (NA); and Place of negation -- that is, in the third premise or conclusion (PON).

area in Lebanon. Three of the schools were for girls, one was for boys, and one was co-educational. Two schools were lower middle class and three were upper middle class. The subjects were bilingual with Arabic as the native language and English as the foreign language.

Statistical Analysis. A stepwise multiple regression technique was used. "The criterion variable was proportion correct. A separate analysis was made using structural variables for each grade alone and for all grades combined in each of the two languages. A second analysis was made using both structural variables and linguistic variables in each of the two languages for each grade and all grades combined. The significance of a variable was determined by the F ratio associated with the regression coefficient at the last step of the analysis."

4. Findings

The four structural variables accounted for 43% to 60% of the variance for the various grade levels using the Arabic version. These variables accounted for 33% to 63% of the variance for the grade levels using the English version of the test. The variable Decidability was significant for each grade level. Overgeneralization was significant for grades 8, 9, and 10 but not for grades 6 and 7. The variable Type of Logic was significant only for grade 8, English version. Length of sequence was not significant for any grade level in any language.

The increase in variance of proportion correct accounted for by including linguistic variables was 21%, 20%, 16%, 17%, and 19% in Arabic for grades 6 through 10 respectively, and 8%, 6%, 10%, 16% and 12% for the same grades in English.

Decidability was the most important single variable for each grade, for all grades combined, and in each language. The significance of the variable Overgeneralization seemed to be related to the linguistic setting in which the variable was cast.

Some linguistic variables were significant for both Arabic and English. The variable PON was significant for most grades in both Arabic and English.

5. Interpretations

Linguistic variables in a foreign language seem to be a more important predictor of item difficulty for older children than for younger children in that age range 11 to 16 years. This does not appear to be true for the native language. Perhaps "syntactical structures reach stability in a foreign language later than those of a native language."

Given that the variable D was significant for all grade levels, it seems that there is not a "developmental trend in the age range 11-16 years

It appears that the ability of students in grades six through ten to recognize valid principles is not a unitary ability which develops with age and levels off at a fixed age. The determinations of the validity of inference patterns is influenced by certain linguistic cues as well as the formal logic involved.

Critical Commentary

The investigator conducted a well-conceived study that focused on important variables in analyzing students' ability to deal with inference patterns. Considerably more information was provided in the article than was possible to report in this abstract. Apparently the investigation is part of a series of investigations dealing with students' understanding of inference patterns.

It would have been helpful to have seen a specific test item but none was given. Further, it would be interesting to know how many items the students were getting correct. Such information was not provided and hence the reader cannot determine just "how well" students understand inference patterns.

The linguistic variables were defined in basically quantitative terms (e.g., the number of predicates with possessive nouns in the item). Subsequent investigations might consider qualitative aspects of the language. For example, the proposition, "If Samir's car is a police car, then it is black," could be presented in a form which does not utilize the "If-then" format, e.g., "Since Samir's car is a police car, it must be black," or "The car is black because it is Samir's car and it is a police car." Words such as "must," "since," "because," and "provided that" could be used appropriately to indicate the same implication. Many of the linguistic schemes for conveying implications in the classroom do not employ the "If-then" format.

Finally, it might be interesting to follow up selected students with interviews to gain more in-depth knowledge on ways that the students dealt with inference patterns.

NUMBER PATTERNS: DISCOVERY VERSUS RECEPTION LEARNING. O'Brien, Thomas C.; Shapiro, Bernard J. Journal for Research in Mathematics Education, v8 n2, pp83-87, March 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gail Spitler, The University of British Columbia.

1. Purpose

The purpose of the study was to compare the efficacy of two types of discovery learning with reception or expository learning as measured by an immediate posttest, a retention test, and a transfer test.

2. Rationale

The study was undertaken to examine the effects of discovery versus reception learning under carefully defined and, hence, replicable conditions. The authors correctly point out that lack of replicability has been a fault of many studies related to discovery learning. The authors also relate the research to Piagetian theory, emphasizing the importance of problems and problem-solving (as opposed to exercises and drill) and the importance of social interaction in learning.

3. Research Design and Procedure

Forty-five full-time elementary or secondary school teachers enrolled in an introductory research methodology course, a requirement of the master's degree program in a university-based school of education, were randomly assigned to one of three treatment groups. These groups were (a) discovery-I, (b) discovery-II, and (c) reception (interchangeably referred to as expository).

All treatments involved (a) an initial learning period of 30 minutes followed immediately by a 15-minute posttest, (b) a 15-minute retention test administered three weeks later, and (c) a 15-minute transfer test administered four weeks after the initial learning session. The initial learning task and the retention test involved the identification of number patterns in a standard addition grid. The transfer test involved the identification of number patterns in a standard multiplication grid. The subjects in the discovery-I group were randomly assigned to small groups of three subjects each. Following the initial presentation of the addition grid and a discussion of two samples of number patterns, the instructions were given.

The discovery-I group was instructed to find as many patterns as they could while working in groups of three. The discovery-II group found as many patterns as they could, working independently. Both

All groups received a special sheet on which to record the patterns identified.

Immediately following the initial learning period, all groups were given 15 minutes in which to list as many patterns as they remembered and as many new patterns as they could find. They were told to work alone without referring to their notes.

Three weeks later, the addition grids were again distributed along with the same instructions as described above. Four weeks following the initial treatment, multiplication grids were introduced and the subjects were instructed to find as many patterns as they could working independently for 15 minutes.

4. Findings

The dependent variable was the number of patterns identified by each subject on each test. Table 1 gives the mean and standard deviation for the posttest and the retention test.

Table 1
Addition Grid: Means and Standard Deviations of Number of Patterns Cited

Time	Treatment: Discovery-I (N = 15)	Discovery-II (N = 15)	Expository (N = 15)
Posttest	9.00 (2.34)*	6.80 (1.42)	10.87 (2.82)
Retention	12.27 (3.53)	6.93 1.65)	6.33 (2.15)

*Figures in parentheses are the standard deviations.

Although no significant differences were found across trials, treatment and interaction effects were significant. The Scheffé a posteriori analyses of the group means showed significant posttest differences in favor of the expository group over either the discovery-I or the discovery-II group, with no significant differences between either of the latter. On the retention test, the discovery-I group significantly out-performed the discovery-II group and the expository group. A simple one-way analysis of variance and subsequent Scheffe analyses of the results of the transfer task indicated that the discovery-I group out-performed either the discovery-II or the expository group, with no significant differences between the latter two groups.

their responses to the patterns actually exhibited on the videotape on both the posttest and the retention test. The discovery-I group suggested four incorrect patterns on both the aforementioned tests, while all the patterns offered by the discovery-II group were correct. The results of the transfer task revealed three, three, and five incorrect patterns offered by the discovery-I, discovery-II, and expository groups, respectively.

5. Interpretations

The authors suggest that this research raises several questions. First, reception or expository learning may be far less effective than discovery learning when long-term retention is considered. Second, independent discovery may be less effective than group discovery. Such findings may indicate that individualized instruction has severe limitations.

Critical Commentary

The authors should be congratulated for completing a study of discovery learning that is indeed well-designed, with appropriate statistical treatment. However, they may be overly optimistic about the replication of such a study. If one has a copy of the videotape used as the expository treatment and the specific sample number examples demonstrated prior to the treatments, then clearly the entire study is capable of replication, if such replication would be of value. However, without said tapes and examples, replication is questionable. In fact, the content of the videotape raises several interesting questions. Is it possible that in reality, the two discovery groups actually acquired techniques for looking for number patterns while the expository group was only finding patterns demonstrated in a random fashion? If so, then the two discovery groups should be expected to out-perform the expository groups, especially on a transfer test. When one considers the broad question of developing related experiments, this study leaves the reader in a quandary -- what guidelines were used to develop the treatments, especially the expository treatment? How would a related study be designed if it involved other content areas? Again, it should be noted that the experimenters fully acknowledge the severe limitations of this study and correctly identify many of the related questions, such as, under what conditions should discovery learning versus expository learning be used?

However, it is this reviewer's opinion that this study suffers from a more important limitation: lack of a conceptual basis. Research related to discovery learning is in dire need of conceptual frameworks that define the treatment differences so that clearly related studies can be completed and compared. Without such conceptual frameworks or models, one cannot develop related experiments that examine different time spans, different content areas, and alternative transfer conditions

THE CONTEXT OF IN-SERVICE EDUCATION. Osborne, Alan; Bowling, J. Michael. An In-Service Handbook for Mathematics Education (Alan Osborne, Ed.). Reston, Virginia: NCTM, pp12-58, 1977.

SUPERVISORS AND IN-SERVICE EDUCATION. Osborne, Alan; Bowling, J. Michael. An In-Service Handbook for Mathematics Education (Alan Osborne, Ed.). Reston, Virginia: NCTM, pp59-71, 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by William E. Geeslin, University of New Hampshire.

1. Purpose

These articles report on two national surveys, one of public school teachers and one of public school mathematics supervisors, which were intended to reflect opinions on the present state of in-service education and the desired state of in-service education.

2. Rationale

Historically, changes in teacher background variables have been accompanied by changes in in-service programs. As teachers become better trained and continue in the profession longer, a different type of in-service programming may be desirable and necessary. "The changing nature of mathematics teachers--their professional aspirations, background, and experience--is the largest factor of change that needs to be accounted for in considering in-service education for the future." The supervisor may be a determining factor in providing a program that serves teachers' needs.

3. Research Design and Procedure

A representative sample of public school teachers in the United States which reflected actual proportions of elementary and secondary teachers and of student populations by state, district, and school was selected. A sample of 3964 elementary and 3482 secondary mathematics teachers was obtained by selecting schools randomly. School principals were contacted and given instructions for randomly selecting a teacher within that school. Surveys were returned by 20.7% (N = 821) of the elementary teachers and 41.1% (N = 1431) of the secondary teachers. NCSM supervisors (N = 742) in the United States also were surveyed and 74% (N = 549) responded. The teacher questionnaires consisted of 147 short-answer items and 3 optional open-ended questions. Twenty-one questions provided data on background variables, 138 questions were common to both elementary and secondary forms, and 49 items were used to generate parallel "what-is" and "what-ought-to-be" questions. Supervisors received a similar questionnaire.

Descriptive statistics, correlation coefficients, and chi-square techniques were used. For most statistics a conservative level of significance used due to the large number of significance tests performed.

4. Findings

Most secondary teachers (60.3%) had taken at least eight mathematics courses beyond calculus and only 12.4% had taken no mathematics methods courses. In fact, many (55.9%) held a Master's degree. Elementary teachers had training in mathematics (68.9% had taken 3 or more college mathematics content courses). Only 10.8% of the elementary teachers had taken no mathematics methods courses, while 35.5% had at least a Master's degree. Most (77.9% of the elementary teachers and 83.3% of the secondary teachers) had more than five years experience. Statistics concerning tenure, salary, school setting, average number of students per class, and grade level taught were given. An NCTM journal was read by 31.8% and 61.1% of the elementary and secondary teachers, respectively. The vast majority would choose teaching as a profession again.

Responses revealed that 82.9% and 70.8% of the elementary and secondary teachers, respectively, had participated in an in-service program since September 1, 1973. The most frequent sponsor of in-service programs appeared to be the local school system. Teachers indicated a need for in-service work. A majority of teachers had had positive experiences with in-service education, but many teachers were not satisfied with past in-service education. Common complaints were that past programs did not fit classroom needs and participants were not involved in topic selection. Thirteen complaints were noted by at least 20% of the teachers. The majority (53.7%) of supervisors devoted only 10% of their time to in-service education and only 23.6% indicated that their school spent more then $5 per teacher on in-service programs. A listing of selected programs was provided.

Analyses on nearly all of the "what-is" versus "what-ought-to-be" questions revealed discrepancies between the present situation and the desired situation. In most cases, respondents desired "more" from the in-service program. For example, only 32% of the elementary teachers and 28% of the secondary teachers noted that their school building or school system had an individual responsible for in-service education in mathematics, but a corresponding 84% and 83% felt there ought to be such an individual. Large discrepancies were noted in areas such as follow-up in the classroom after in-service programs, single-topic all-day offerings, programs designed especially for mathematics, workshops on motivation and applications, new teaching methods, use of achievement data in determining needs, and provision of release-time from the classroom to encourage in-service participation.

Secondary teachers who noted the availability of an individual responsible for programming were more positive toward in-service programs, had fewer complaints, and thought in-service attendance should be required. Secondary teachers who have participated in program topic selection are more likely to express satisfaction with in-service. Summer programs were not ice programs was related significantly to twelve variables such as satisfaction, positive in-service experience, background, and complaints. Satisfaction variables were related positively with experience in workshops focusing on computation. The topic of mathematical structure seems not to be part of recent in-service training and teachers with less experience are unlikely to have participated in such a program. Nine possible purposes of in-service education were all considered highly desirable, but learning about new curricula was the most common purpose. Providing release-time from classroom duties was the primary method (and most desirable for teachers) by which schools encouraged participation. Teachers who felt students' enthusiasm for mathematics had declined were less likely to believe achievement should be used as a criterion of effectiveness of in-service education programs.

5. Interpretations

It was believed that non-respondents were more likely than respondents to be negative toward in-service education. Respondents were well trained, experienced teachers who were reasonably satisfied with their profession and employment. In-service programs should not favor elementary over secondary teachers even though elementary teachers appeared more satisfied with programs. Considerable effort must be expended to satisfy the needs and interests of secondary teachers. Teacher's complaints must be considered. Only 4 of 49 discrepancies between the actual and desired states of in-service education failed to reach significance, indicating need for improvement. The usefulness of the content and materials is extremely important since it affects teachers' perceptions of power, reward, and gripes. Important topics are "computation, motivation, applications, learning difficulties, diagnosis, evaluation, articulation, remediation, and attitudes." The components of enthusiasm and entertainment are necessary. In-service education should be directed toward specific disciplines such as mathematics teaching. Supervisors must use effective political action to establish and maintain in-service programs with "protected dollars." Supervisors must be extremely careful in allocating resources.

Critical Commentary

In-service education is clearly an important topic and the articles with associated appendices contain a plethora of useful information, only a small portion of which could be included here. The articles should be read by everyone involved in mathematics in-service program planning. Nonetheless, the naive research reader should use extreme caution in relying on the information presented. Reporting research in popular prose format is a difficult task at best. A very fine line exists between using non-technical language and interpretation unsupported by data. Correlational and chi-square techniques tempt writers to make inappropriate "causation" remarks. Osborne and Bowling succumbed to these hazards on occasion as well as overemphasized their personal happiness/unhappiness tantly that I would use similar casual statements in proposal justifications or NCTM talks.

In essence, only descriptive results of opinion information were presented. A historical account of in-service education does not suffice for a "theoretical framework." Sampling and survey techniques appeared sound. Were state supervisors included in the sample? Problems with some answer choices were acknowledged. The use of terms such as variance in both the common language sense and the statistical sense should be avoided (p. 35). At times actual responses (p. 42) and rankings were unclear (p. 43). The low response rate of elementary teachers and dependence on principals in the sampling process are causes for concern. The authors engaged in considerable unsupported supposition about the respondents It is not clear that in-service programs evolve through attempts "to better serve the teaching and learning of mathematics." Reliability could have been more thoroughly discussed. What-ought-to-be questions may result in "motherhood" responses. Shuffling between two articles and three appendices was difficult. Finally, an empirical connection between teacher opinion and teacher behavior/student learning are necessary future steps.

THE QUALITY OF STANDARDIZED HIGH SCHOOL MATHEMATICS TESTS. Petroski, Joseph M. Journal for Research in Mathematics Education, v9 n2, pp137-148, March 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James T. Fey and William J. Engelmeyer, University of Maryland.

1. Purpose

The objective of the study was to examine critically the content and technical qualities of mathematics tests intended for high school students in four areas: general mathematics, algebra, geometry, and applied mathematics.

2. Rationale

Standardized mathematics tests are used extensively. Indeed, the use as a tool in important instructional decision-making situations is increasing. However, criticism of the content validity, technical adequacy, and interpretability of many tests is widespread, suggesting the need for careful analysis of commonly used tests. The present study is part of a larger effort based at the UCLA Center for the Study of Evaluation.

3. Research Design and Procedure

The investigators collected over 500 specimen tests and supporting technical information for mathematics tests that are available and appropriate for use in grades 9-12. After categorizing each test, or identifiable sub-test, according to content and grade level of usage, each test was rated for quality on 39 criteria. The criteria of quality fell into four broad areas: validity, appropriateness for examinees, administrative usability, and normed technical excellence. Rating was done by a pair of evaluators with graduate training in education or psychology. The evaluators worked independently, basing their ratings on inspection of the tests and available supporting information provided by the test publishers.

4. Findings

The results of test ratings are presented in a variety of summary tables, the basic format being percentage of tests which fit into each rating category on each rated dimension. The overall impression from inspection of these tables is that the vast majority of available tests are weak in many crucial aspects of test quality. Very few of the tests have content based on empirical procedures for item selection, measures of concurrent validity, or measures of predictive validity. Most tests have low or unreported measures of reliability, norm groups that are not nationally representative, and low ratings on usefulness for educational decisionsimplicity of raw score conversion to a common interpretable score. Moreover, a very high percentage of the tests used reporting scales, such as grade equivalents or mental ages or novel scales, that have been severely critized recently.

5. Interpretations

The investigator's general conclusion is that current mathematics tests are deficient in many basic aspects of test quality, confirming the criticisms leveled at standardized tests from many quarters. In addition to the standard criteria of validity, reliability, and interpretability, the available mathematics test instruments seem of limited value in guiding educational decisions about individuals or groups. They are also seriously flawed by the limited range of outcomes measured, making the problem of curricular evaluation and/or comparison extremely difficult. There is a suggestion to follow the NLSMA model of parallel curriculum and test development in order to overcome the latter shortcoming in evaluation.

Critical Commentary

The investigator's rationale for examining test quality cited three common criticisms of standardized tests: (1) not covering some important components of ability; (2) overtesting other components of ability; and (3) testing a unitary trait, rather than a complex set of skills. Unfortunately, however, the study that was based on this problem definition does little to confirm or deny the criticisms and less to suggest viable solutions. The rating staff consisted primarily of people unqualified to make the professional judgments necessary in assessing the extent to which current tests measure (in appropriate balance) the full range of important school mathematics objectives. Instead, we are offered a collection of technical quality ratings based on information supplied by test developers in their typical packages of commercial information. As a result, even if the 39 indicators had suggested that available tests are of high technical quality, the fundamental controversy over validity would remain.

Because the investigator reviewed all available standardized mathematics tests, the investigation was thorough. However, it is difficult to extrapolate the findings to well-known tests. As the investigator indicated, accurate estimates should be compiled of the frequency of test purchase in order to separate major and minor instruments.

There is a degree of ambiguity in trying to interpret the tables. Since all evaluations are recorded in tabular form, it is impossible to review the evaluation of any single test. A reference in the article does imply that information pertaining to the evaluation of individual tests may be available.

INFORMATION MATCHING WITHIN AND BETWEEN AUDITORY AND VISUAL SENSE MODALITIES AND MATHEMATICS ACHIEVEMENT. Sawada, Daiyo; Jarman, R. F. Journal for Research in Mathematics Education, v9 n2, pp126-136, March 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Peggy A. House, University of Minnesota.

1. Purpose

The study investigated the hypothesis that there is a predictive relationship between sense modality matching abilities and mathematics achievement in fourth-grade boys. Of particular interest to the investigators were auditory-auditory (A-A), auditory-visual (A-V), visual-auditory (V-A), and visual-visual (V-V) matching abilities. Specifically, the investigation focused on four hypotheses:

(a) There are significant correlations between mathematics achievement and modality matching abilities.

(b) There are significant variations in these correlations across three IQ ranges.

(c) The magnitudes of the correlations involving mathematics and their variation over the IQ ranges are as great or greater than the correlations involving reading achievement.

(d) The intercorrelations of the modality matching abilities show significant variations across the three IQ groups.

2. Rationale

The concern of the study was auditory and visual sensory modality matching in which a stimulus was presented in one sense modality and the subject was asked to identify a second pattern in the same or the other sense modality as being either alike or different. For example, a pattern of beeps (auditory stimulus) would be matched with a pattern of dots (visual). Such matching ability is presumed to be important in school mathematics where many learning experiences require the pupil to perform higher-order matching. For example, the teacher says "four" and the pupil is to circle four objects in a picture. While research in reading has supported the conclusion that A-V matching ability significantly predicts reading achievement, research reported in mathematics education has not included modality matching ability either as a central concern in the design of treatments intended to improve perceptual ability or as a consideration for statistical analysis reading, there is a predictive relationship between sense modality matching abilities and mathematics achievement.

3. Research Design and Procedure

The Ss were 180 fourth-grade boys from 19 public schools in middle socioeconomic areas of Edmonton, Alberta. Ss with "identifiable family, personal or disability problems" as judged by principals, teachers, or cummulative records were eliminated from the population. The Ss chosen represented a stratified random sample of 60 low- (IQ 71-90), 60 middle- (IQ 91-110) and 60 high- (IQ 111-130) ability pupils as measured by the verbal IQ score of the Lorge-Thorndike Intelligence Test.

Four tests (A-A, A-V, V-A, V-V) were administered to the Ss. Each test consisted of 35 items of same-different matching, the first five items being practice items only. The reported KR-20 reliabilities of these tests were from .60 to .84, a range judged by the investigators as "not exceptionally high, but adequate." The auditory patterns employed 1000 cycle tones with variations caused by patterns of short or long pauses. Visual patterns consisted of displays of variously spaced dots. Tests were administered to Ss in groups of four to six. In order to avoid transfer effects from taking the four tests in succession, the investigators used a 4 x 4 Latin square design to vary the sequence in which the tests were administered.

Four achievement measures were selected for testing the hypothesized correlations. Three measures of verbal achievement were subtests of the Stanford Achievment Test, Form W (1965): word meaning, paragraph meaning and word study skills. Mathematics achievement was measured by a test developed by the Edmonton Public School System to assess "understanding of basic arithmetic operations including number lines, relationships, numeration, and geometric figures." The mathematics test has a KR-20 reliability estimate of 0.87. Data analysis consisted of correlations between the four achievement measures and the four modality matching tests for each IQ group.

4. Findings

For the low IQ group, the investigators reported a significant correlation ($r = 0.56$, $p<.01$) between mathematics achievement and A A matching only. For the high IQ group, the correlations between mathematics achievement and each of the four modality matching tests were significant ($r = 0.39$, $p<.01$ to $r = 0.30$, $p<.05$). Mathematics achievement did not correlate significantly with any of the modality matching tests for the middle IQ range. These results were considered to give support to the first two hypotheses.

The only significant correlations with reading achievement occurred for the low IQ groups: between word meaning and A-V matching ($r = .29$,

$p<.05$) and between word study skills and V-A matching ($r = .25$, $p<.05$). That the magnitudes of these correlations are less than the significant correlations involving mathematics was cited as evidence in support of Hypothesis C.

Finally, the investigators reported intercorrelations among A-A, A-V, V-A and V-V performance for each IQ level. The correlations of A-A performance with the other matching abilities ranged from .18 to .49 and generally increased with increasing IQ. All other correlations remained relatively constant over IQ groups. Hypothesis D gained some support from this changing relationship of A-A to the other modality matching abilities.

5. Interpretations

The relationships of modality matching abilities and mathematics achievement appear to depend upon the type of matching and on the IQ of the pupil. As IQ increases, A-A matching appears to become more highly correlated with other modality matching abilities. This may enable higher IQ pupils to compensate for inability in some A-A tasks by converting to other modality combinations. This further suggests that instruction which depends principally on listening must be used carefully in teaching low IQ pupils, in particular when instructions given in the auditory mode must be related to learning tasks requiring the use of A-V, V-A, or V-V processing of information. The changing nature of the interrelationships among modality matching abilities with changing IQ levels also suggests that modality matching ability may be a useful variable to include in ATI experiments.

Critical Commentary

The study is based on an interesting paradigm and it is significant in raising important questions about the pupil's ability to translate within and between sensory modalities. The preliminary findings represented by the present study suggest a potentially fruitful area of research. In particular, results of modality matching studies can contribute to an understanding of the learning processes in a multisensory learning environment such as the mathematics laboratory.

The pattern of significant correlations between mathematics achievement and modality matching ability for high IQ pupils raises the question of a possible relationship with the Piagetian developmental level of these pupils. It is possible that the higher-ability fourth graders have more fully attained a higher operational level that facilitates their modality matching performances while lower-ability pupils may still be to some extent preoperational and less able to abstract matching patterns.

The sample of fourth-grade boys is, of course, very limited, and extension of similar investigations to other ages and both sexes would be useful. In considering future studies, several questions might be considered:

1. The IQ measure used to stratify Ss is verbal IQ, yet the achievement measure of primary interest is mathematics. Would the results be similar if Ss were stratified on a measure of mathematical ability such as quantitative reasoning or spatial perception?

2. Potential subjects with "identifiable family, personal or disability problems" were eliminated, but the study reports neither the operational definitions of these problems nor the criteria by which the eliminations were determined. Neither does it establish the fact that the sample is not biased as a result of such exclusions.

3. It is not clear what is measured by the mathematics achievement test selected for the study. Perhaps several mathematics measures are needed to differentiate among computational ability, quantitative reasoning, or similar variables. This might lead to a clearer delineation of possible relationships between aspects of mathematics achievement and modality matching abilities.

4. It is difficult to interpret the investigators' comparisons of the magnitudes of significant correlations involving mathematics and reading. True, the magnitudes of the five significant mathematics correlations exceed, at least slightly, the magnitudes of the two significant reading correlations, but the comparisons were made across IQ levels and types of sensory modality tests. It might be more meaningful, for a given IQ level, to focus on pairwise comparisons of the correlation of mathematics achievement with a given modality matching type (for example, A-A) and the correlation of reading achievement with the same modality matching type.

There also are questions arising from the investigators' conclusion that A-A matching ability is more highly correlated with other modality matching abilities as IQ increases. This is true for the correlations between A-A and A-V. However, except for this one case, all intercorrelations among modality matching abilities are reported to be highest within the middle IQ range where none of the correlations between achievement and modality matching ability reached significance. This weakens the suggested hypothesis that high IQ pupils compensate for A-A difficulties by switching to other sense modalities.

The investigators have indicated several lines of future research which they intend to pursue. The preliminary results reported in the present study warrant the encouragement of those efforts.

INQUIRY STRATEGY, COGNITIVE STYLE, AND MATHEMATICAL ACHIEVEMENT. Scott, Norval C. Journal for Research in Mathematics Education, v8 n2, pp132-143, March 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Joe Dan Austin, Emory University.

1. Purpose

This study investigated whether teaching science using an inquiry method had any longitudinal effects on students' mathematical achievement.

2. Rationale

Suchman and others used an inquiry strategy in science teaching. Research has indicated that the analytical aspects of students' cognitive style was significantly improved by this training. Further, this improvement seemed to persist over time. The "perceptual" approach to cognitive style has an analytical component. Research has indicated a strong relationship between this analytical component and mathematical achievement. Therefore it is reasonable to expect that students exposed to inquiry become more analytical. Thus they should do better in mathematics than students not exposed to inquiry teaching.

3. Research Design and Procedure

Two groups of high school students who had completed high school geometry or algebra or both were selected for this study. The Inquiry Group (N = 39) had at least one year of sixth- or seventh-grade science taught using an inquiry method. The Comparison Group (N = 40) had had a similar course but taught in a traditional manner. Scores on the California Test of Mental Maturity (CTMM) indicated the two groups were initially comparable on ability level.

The inquiry-oriented science class used class investigations of "discrepant events". Students could ask the teacher only questions answerable by a "yes" or "no" response. Questioning continued until the students felt that they could explain the experimental situation and results. Some of the conceptual ideas were considered common to geometry or algebra as an explanation or proof was required to explain why the results occurred as they did.

All evaluation was done five years after the sixth- or seventh-grade science course. The two groups were compared on analytical "conceptual" style. The Sigel Cognitive Style Test (SCST) was used to identify each student's analytical "conceptual" style. The two groups were compared on both algebra and geometry course grades. The correlations between analytical "conceptual" style and course grades were also investigated.

4. Findings

A chi-square analysis indicated that analytical responses were significantly ($p < .01$) higher for the Inquiry Group. The Inquiry Group had significantly higher ($p < .05$) course grades in both geometry and algebra as indicated by t-tests. None of the correlations between analytic responses and mathematics grades was significant ($\alpha = .20$).

Students were asked what they recalled about their sixth-grade science class. Seventy-eight percent of the Inquiry Group students recalled problem-solving activities, while only five percent of the Comparison Group students did. A questionnaire on the effects of inquiry was sent to 26 Inquiry Group students. The responses ($N = 17$) indicated stronger support for the effect of inquiry teaching in geometry than in algebra.

5. Interpretations

The implications were that this study

(a) "...has raised some questions about the validity of the term 'analytical' when referring to a stylistic disposition." and

(b) "...indicates that senior high school mathematics competency was enhanced by this specific inquiry program." (p. 143)

The author also suggests the need to investigate "field independency" and senior high mathematics grades of students exposed to this inquiry program.

Critical Commentary

This study investigates an interesting question on the relationship between a science teaching method and mathematical achievement. The paper is clearly written both as to the procedures and analysis used. While the number of subjects is small, the author acknowledges this to be a pilot study and seems careful about generalizing from the results.

This reviewer had several questions about the paper. The first concerned the initial equivalence of the two groups. Was the CTMM test in 1966 administered before the inquiry-oriented science course? While this seems likely it is not stated in the paper. Also, it would have been desirable to have a measure of students' initial mathematical abilities to be more comfortable with the initial comparability of the groups. Another question concerns the entire section on students' recall about their sixth-grade science class. Why were only 26 of the 39 Inquiry students given the questionnaire? Why -- with so few students -- were only 17 responses received? This plus the absence of any statistical tests make these results somewhat suspect.

In spite of these questions the main results seem valid. If the results can be replicated, it would seem logical to try to adapt the teaching method to mathematics instruction. Would this have a stronger influence on mathematical achievement than the transfer from science instruction?

THE UNDERSTANDING OF SIMILARITY AND SHAPE IN CLASSIFYING TASKS. Vollrath, Hans-Joachim. Educational Studies in Mathematics, v8 n2, pp211-224, August 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward M. Carroll, New York University.

1. Purpose

Assuming that similarity can be tested with classifying tasks, the purpose of the study was to provide evidence for answering the following questions:

(a) What are Ss' understandings of the expression "similar" as used in colloquial speech to apply to geometrical figures?

(b) Does the "geometrical context" of the basic set of figures influence the results of the sorting process?

(c) Is there a relationship between age and the results of the sorting problems?

(d) Is there a relationship between sex and the type of sorting solutions?

(e) How do the terms "similar" and "same shape" in the directions influence the results of the sorting?

(f) What is the relationship between the arrangement of sorting tasks and the results?

2. Rationale

The names of many geometrical concepts are used not only in geometry but also in colloquial speech. But as a result of a process of mathematization in geometry, they are restricted by axioms and definitions. The names of some geometrical concepts are connected with images which do not belong to the geometrical concept (e.g., similarity, plane, curve, angle, etc.). Therefore when these names are used in geometry, understanding of the concepts may be hindered because properties may be retained which are not included in the geometrical concept and which may therefore lead to false inferences.

Previous research investigations on similarity (Piaget and Inhelder, 1948), on verbal context in developing understanding the meaning of unknown words (Kaplan, 1950), on judgments of attributes (Postman and Page, 1947), and on free-object-sorting tests (Gardner, 1953) were used by the author to develop the rationale for the study.

3. Research Design and Procedure

Three experiments (N = 110, N = 50, N = 30) were conducted using students ranging in ages from 8 years to 19 years, randomly selected from different classes in different schools in the northern part of Bavaria, Federal Republic of Germany, during the autumns of 1975 and 1976. Six tests were constructed using different sets of paper geometric figures; five of the sets were cut from white paper and one set was cut from checked (graph) paper. Test One included eight plane figures, four round, simple closed curves and four many-sided polygons. Test Two included 15 plane figures, three each of circles, squares, triangles, rectangles, and trapezia. Test Three consisted of 11 plane figures: five squares and six rectangles. Test Four contained nine plane figures (rectangles). Test Five and Test Six included six rectangular plane figures with side-length ratios of 1:2, 1:3, and 1:4.

In geometry similarity is a relation between figures (point set). A Figure $\underline{F}_1$ is similar to a Figure $\underline{F}_2$ if there exists a similarity transformation $\underline{S}$ such that $S(\underline{F}_1) = \underline{F}_2$. It follows that one rectangle is similar to another if and only if the ratios of the sides are equal. The sequence of sorting tasks was designed to guide the Ss to discover side-length ratios of rectangles.

In each test all of the figures were placed on a table unordered. In Experiment I, for each of the six tests, the 110 Ss were directed to "Put all similar figures in a heap." In Experiment II, for each of the six tests, the 50 Ss were directed to "Put all figures of the same shape in a heap." In Experiment III, only Text 6 was administered to 30 Ss who were directed to "Put all similar figures in a heap." There were no further comments or aids after the directions were given for each test in each experiment. After each test the results of the sorting were noted, the materials were collected and the next test was offered. The results were classified by the frequency of occurrence. The solutions in Experiment I and Experiment II were compared. The chi-square test was employed to test the hypotheses that there was no difference between the sorting results of the two groups in Experiments I and II respectively, for each of the six tests.

4. Findings

In each of the three experiments, a rather large number of solutions ($10 \leq n \leq 20$) was found in each test. There was no solution among the whole sample which took account of the attribute of rectangles having the same side-length ratio. Thus no one discovered the mathematical similarity as a principle of classifying the geometric figures.

When the results of Experiment I and Experiment II were compared, it was found that:

(a) A wide range of different solutions resulted from each request.

(b) Varying the terms "similar" and "same shape" in the manner tested did not lead to an overall .olution with respect to geometrical similarity of rectangles. However, in two of the subtests, Test Three ($p < 0.05$) and Test Four ($p < 0.001$), there were significant relations between the type of request (similar, same shape) and the type of solutions. On the other four tests, the hypothesis that there was no difference between the results in the two groups could not be rejected.

(c) There was a significant influence ($p < 0.001$) of the preceding tests on the type of solution and the arrangement of the tests, as was shown when Test Six was given in isolation.

(d) Sex and age did not seem to influence the solution.

5. Interpretations

The request to put together similar figures is understood in many different ways. All sorting tests might be explained by the personal styles of experiences. The Euclidean solution was obviously not noticed in these experiments. Although "similar" in the Euclidean sense and "of the same shape" indicate the same geometrical relation, they are understood in different ways when used in sorting problems. "Similar" seems to tend more to attributes of the figure, whereas "same shape" tends more to the "Gestalt." Teaching strategies for geometrical concepts which also have a meaning in colloquial speech should take into consideration the fact that the mere knowledge of the name does not include the full geometrical understanding of the concept. It is necessary to standardize the understanding within the class by stating a definition.

Critical Commentary

There should be more investigations of this nature, exploring how the names of mathematical concepts which are also used in colloquial speech affect the development of learning the mathematical concept. The issue is important for the learning and teaching of geometry in elementary as well as secondary school mathematics. The problem has previously been investigated by Piaget and Minski, but the author fails to cite how his findings were compared to those previously known results.

The report of the study is incomplete in the following respects:

1) The method used to administer the tests is unclear: whether the tests were administered to large groups, to small groups, or to individuals is unstated.

2) The author alluded to results expected for the sorting

solutions from students who had not been taught formal geometry, but failed to indicate how many of his Ss (ages ranging from 8 years to 19 years), if any, had had a formal course in the subject.

3) The results reported only that there was no relationship of age and sex related to the Ss in the first experiment (N = 110, ages 8 years to 15 years) who were requested to sort "similar" figures. Were similar results found relative to age and sex of the Ss in the second experiment (N = 50, 8 years to 19 years) who were requested to sort "same shape" figures?

The concept of similarity is taught in the elementary and junior high school informal geometry. Emphasis is on the "same shape" with little attention given to the side-length ratio or to the proportional concepts. Maybe we should find out how elementary school teachers would respond to the ("same shape", "similar") requests in these sorting tasks.

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